

Shell plus pairing effect arguments for cluster preformation at the nuclear surface in cold fission

D. N. Poenaru* and R. A. Gherghescu^{1,*}

¹*Horia Hulubei National Institute of Physics and Nuclear Engineering (IFIN-HH),
P.O. Box MG-6, RO-077125 Bucharest-Magurele, Romania and
Frankfurt Institute for Advanced Studies, Johann Wolfgang Goethe University,
Ruth-Moufang-Str. 1, D-60438 Frankfurt am Main, Germany*

(Dated:)

In 1928 G. Gamow as well as Condon and Gurney gave the first explanation of alpha decay as a quantum tunnelling of a preformed particle at the nuclear surface. Soon after experimental discovery in 1984 by Rose and Jones of cluster radioactivity, confirming earlier (1980) predictions by Sandulescu, Poenaru and W. Greiner, a microscopic theory also explained the phenomenon in a similar way. Here we show for the first time that in a spontaneous cold fission process the shell plus pairing corrections calculated with Strutinsky's procedure may give a strong argument for preformation of a light fission fragment near the nuclear surface. It is obtained when the radius of the light fragment, R_2 , is increased linearly with the separation distance, R , of the two fragments, while for $R_2 = \text{constant}$ one gets the well known two hump potential barrier.

PACS numbers: 25.85.Ca, 24.75.+i, 21.10.Tg, 27.90.+b

In 1928 G. Gamow [1] as well as Condon and Gurney [2] gave the first explanation of alpha decay as a quantum tunnelling of a preformed particle at the nuclear surface. Soon after experimental discovery in 1984 by Rose and Jones [3] of cluster radioactivity, confirming earlier (1980) predictions by Sandulescu, Poenaru and W. Greiner [4], a microscopic theory [5] also explained the phenomenon in a similar way. Here we show for the first time that in a spontaneous cold fission process [6] the shell plus pairing corrections calculated with Strutinsky's procedure [7] may give a strong argument for preformation of a light fission fragment near the nuclear surface. It is obtained when the radius of the light fragment, R_2 , is increased linearly with the separation distance, R , of the two fragments, while for $R_2 = \text{constant}$ one gets the well known two hump potential barrier.

Among the almost 2450 nuclides (nuclear species) known up to now only 288 are stable to occur primordially: their half-lives are comparable to, or longer than the Earth's age (4.5 billion years), hence a significant amount survived since the formation of the Solar System. The metastable nuclides are decaying toward the stable ones.

The first informations about nuclei have been obtained in 1896, when Antoine Henri Becquerel discovered a "mysterious" radiation of a uranium salt (potassium uranyl sulfate) which was bent by a magnetic field. The term radioactivity was coined by Marie Curie. Together with her husband, Pierre Curie, they discovered radium (symbol Ra) and polonium (Po), which possess a million times much stronger radioactivity.

Ernest Rutherford (ER) gave the names α (^4He nuclei), β (electrons) and γ (electromagnetic radiation with frequencies of 3×10^{19} Hz or higher and wavelengths of 10^{-11} m (10 pm) or lower). Three of the four funda-

mental forces (strong, weak, and electro-magnetic) are responsible for them. They are produced by the decay of excited nuclei of radioactive elements. Gamma rays can penetrate through several centimeters of lead and large doses of them are harmful. From scattering experiments (1911) ER deduced that atomic particles consisted primarily of empty space surrounding a central core called nucleus. He transmuted one element into another, elucidated the concepts of the half-life and decay constant. By bombarding nitrogen with α -particles produced oxygen. The atomic nucleus was discovered around 1911.

After 1928 the microscopic theories of α decay have been developed, see e.g. [8]. The theory was also extended to explain cluster decays [5]. Simple relationships are also very useful [9, 10] to estimate the half-lives.

The liquid drop model (LDM) was introduced by Sir John William Strutt, Lord Rayleigh. His book *Theory of Sound*, was published in 1878. Niels Bohr applied the LDM to Nuclear Physics [11]. It was used by Lise Meitner and her nephew O.R. Frisch [12] to explain the *induced fission* discovered by Otto Hahn and Fritz Strassmann, who identified by chemical means among the fission fragments a barium isotope [13]. N. Bohr and J.A. Wheeler [14] published a theoretical paper, based on his LDM; they showed that fission was more likely to occur with ^{235}U than ^{238}U . An interesting historical account of the discovery of induced fission was written in 1984 by the famous Edoardo Amaldi, former member of the Enrico Fermi's (EF) team who did the first experiment two years before Otto Hahn, but was wrong in interpreting the data. Even a genius like EF was a human being, hence could be sometimes wrong. The energetic (nuclear power plants) and military applications of the induced fission changed completely our world.

Spontaneous fission was discovered in 1940 by G.N.

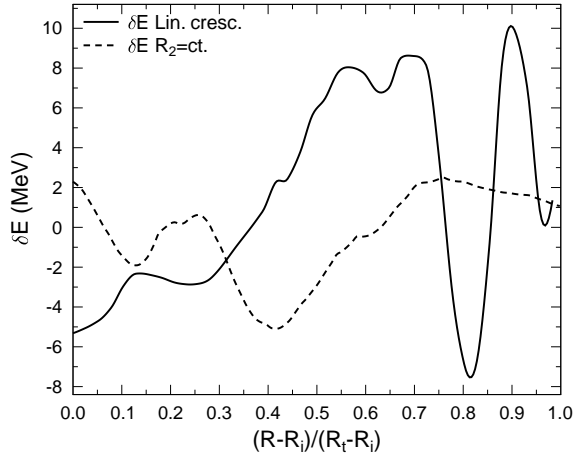


FIG. 1. Comparison of absolute values of shell and pairing correction energies for symmetrical fission of ^{282}Cn with R_2 constant (dashed line) and linearly increasing R_2 (solid line).

Flerov and K.A. Petrzhak [15]. Usually the fission fragments are deformed and excited; they decay by neutron emission and/or γ rays, so that the total kinetic energy (TKE) of the fragments is smaller by about 25-35 MeV than the released energy, or Q-value. The asymmetric mass distributions of the fission fragments and the spontaneously fissioning shape isomers [16] could not be explained until 1967, when V.M. Strutinsky reported [7] his macroscopic-microscopic method. His calculations gave for the first time a two hump potential barrier. Shape isomers occupied the second minimum.

Besides α , β , γ decay and fission there are other types of nuclear disintegrations sometimes referred to “exotic decay modes” as beta-delayed particle emissions, particle-accompanied fission (or ternary fission), fissioning shape-isomers, proton radioactivity, heavy particle radioactivities (HPR) [17, 18], etc. A brief presentation, at a level of non-specialist, of the large diversity of nuclear decay modes may be found in the Ref. [19].

Superheavy nuclei with atomic numbers $Z = 104 - 118$ are produced by fusion reactions [20, 21]. The simplest way to identify a new superheavy element synthesized in such a way is to measure its α decay chain, down to a known nuclide. Sometimes this is not possible since its main decay mode could be spontaneous fission. For atomic numbers larger than 121 cluster decay may compete as well [22]. Among the many theoretical papers in this field one should mention [23] and [24, 25].

We reported [26] results obtained within macroscopic-microscopic method using cranking inertia [27] and the best two-center shell model [28] in the plane of two independent variables (R, η) , where R is the separation distance of the fragments and $\eta = (A_1 - A_2)/A$ is the mass asymmetry with A, A_1, A_2 the mass numbers of

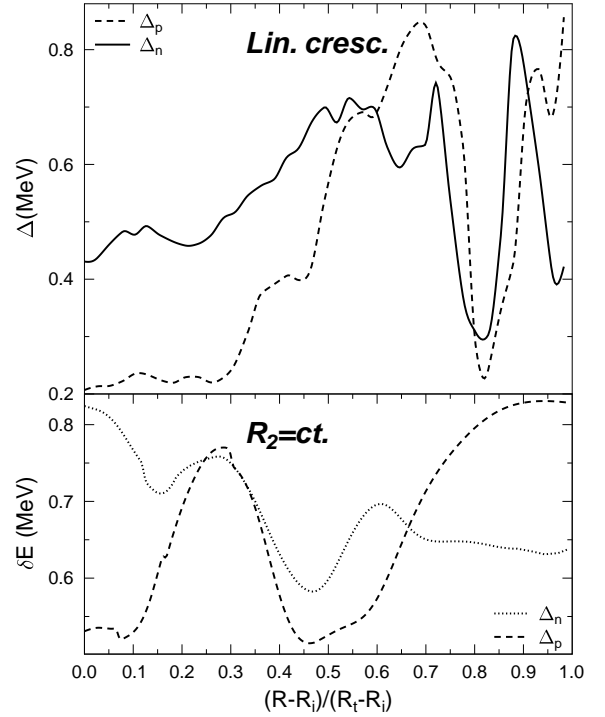


FIG. 2. Solutions of BCS equations for symmetrical fission of ^{282}Cn with linearly increasing R_2 (top) and constant R_2 (bottom). The gap for protons and neutrons do have a similar behaviour with that of the shell corrections.

the parent and nuclear fragments. Phenomenological deformation energy, E_{Y+E} , was given by Yukawa-plus-exponential model [29], and the shell plus pairing corrections, $\delta E = \delta U + \delta P$ are based on the asymmetric two center shell model (ATCSM). This time we give more detailed arguments for the neighboring nucleus ^{282}Cn . The deep minimum of total deformation energy near the surface is shown for the first time as a strong argument for cluster preformation.

An outline of the model was presented previously [26]. Here we repeat just few lines. The parent $^A Z$ is split in two fragments: the light, $^{A_2} Z_2$, and the heavy one, $^{A_1} Z_1$ with conservation of hadron numbers $A = A_1 + A_2$ and $Z = Z_1 + Z_2$. The corresponding radii are given by $R_0 = r_0 A^{1/3}$, $R_{2f} = r_0 A_2^{1/3}$, and $R_{1f} = r_0 A_1^{1/3}$ with $r_0 = 1.16$ fm. The separation distance of the fragments is initially $R_i = R_0$ and at the touching point it is $R_t = R_{1f} + R_{2f}$. The geometry for linearly increasing R_2 from 0 to $R_{2f} = R_e$ is defined by:

$$R_2 = R_{2f} \frac{R - R_i}{R_t - R_i} \quad (1)$$

According to the macroscopic-microscopic method the total deformation energy contains the macroscopic Yukawa-plus-exponential (Y+EM) term and the shell

plus pairing corrections

$$E_{def} = E_{Y+E} + \delta E \quad (2)$$

In units of $\hbar\omega_0^0 = 41A^{-1/3}$ the shell corrections are calculated with the Strutinsky procedure as a sum of protons and neutrons contributions

$$\delta u = \delta u_p + \delta u_n \quad (3)$$

One obtains a minimum when there are important bunchings of levels (high degeneracy of the quantum state: the same energy corresponds to several states).

The BCS [30] theory was first introduced in condensed matter in order to explain the superconductivity at a very low temperature. It was extended to nuclei for explanation of the pairing interaction, see e.g. [27]. By solving the BCS system of two equations, with two unknowns, we find the Fermi energy, λ , and the pairing gap Δ , separately for protons and neutrons. The total pairing corrections are given by

$$\delta p = \delta p_p + \delta p_n \quad (4)$$

and finally the total shell plus pairing corrections in MeV

$$\delta E = \delta U + \delta P \quad (5)$$

Pairing correction is in general smaller in amplitude and in antiphase with shell correction; it has an effect of smoothing and reducing the total shell plus pairing correction energy. The experience of using Strutinsky's method, gained by several nuclear scientists (e.g. S. Bjørnholm), was also successfully employed to study shell effects in atomic cluster physics and nanotechnology.

The inertia tensor [27] is given by

$$B_{ij} = 2\hbar^2 \sum_{\nu\mu} \frac{\langle \nu | \partial H / \partial \beta_i | \mu \rangle \langle \mu | \partial H / \partial \beta_j | \nu \rangle}{(E_\nu + E_\mu)^3} (u_\nu v_\mu + u_\mu v_\nu)^2 \quad (6)$$

where H is the single-particle Hamiltonian allowing to determine the energy levels and the wave functions $|\nu\rangle$; u_ν^2 , v_ν^2 are the BCS occupation probabilities, E_ν is the quasi-particle energy, and β_i, β_j are the independent shape coordinates.

For spherical fragments with R, R_2 deformation parameters the cranking inertia symmetrical tensor will have three components, hence the scalar

$$B(R) = B_{R_2 R_2} \left(\frac{dR_2}{dR} \right)^2 + 2B_{R_2 R} \frac{dR_2}{dR} + B_{RR} \quad (7)$$

or $B = B_{22} + B_{21} + B_{11}$. When we find the least action trajectory in the plane (R, R_2) we need to calculate the three components B_{22}, B_{21}, B_{11} in every point of a grid of 66×24 (for graphics) or 412×24 (for the real calculation) for 66 or 412 values of $(R - R_i)/(R_t - R_i)$ and 24 values of $\eta = (A_1 - A_2)/A$ or R_{2f} .

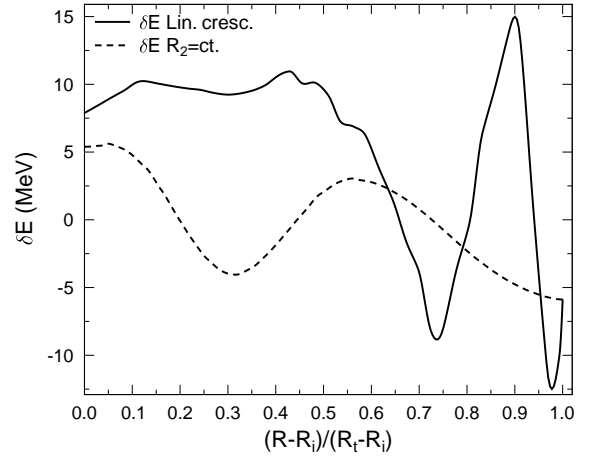


FIG. 3. Comparison of shell plus pairing effects for fission of ^{252}Cf with linearly increasing R_2 and constant R_2 .

We compare in figure 1 the absolute values of shell and pairing correction energies for symmetrical fission of ^{282}Cn with R_2 constant (dashed line) and linearly increasing R_2 (solid line). As expected, the gap for protons, Δ_p , and neutrons, Δ_n , solutions of the BCS system of two equations, in figure 2 are also following similar variations. Deep minima around $(R - R_i)/(R_t - R_i) = 0.82$ are clearly seen in both figures. Similar results are also obtained for heavy nuclei like ^{252}Cf (see figure 3) or ^{240}Pu . At the touching point, $R = R_t$, both kinds of variations of $R_2 = R_2(R)$ are arriving at the same state, hence the shell effects are identical there, as may be seen in figures 1 and 3.

For minimization of action we need not only B_{RR} but also the values of $B_{R_2 R_2}, B_{R_2 R}$ in every point of the above mentioned grid. As expected we obtained a dynamical path very different from the statical one. We could reproduce the experimental value of ^{282}Cn spontaneous fission half-life, $\log_{10} T_f^{exp}(s) = -3.086$.

In conclusion, with our method of calculating the spontaneous fission half-life including macroscopic-microscopic method for deformation energy based on asymmetric two-center shell model, and the cranking inertia for the dynamical part, we may find a sequence of several trajectories one of which gives the least action. Assuming spherical shapes, we found that the shape parametrization with linearly increasing R_2 is more suitable to describe the fission process of SHs in comparison with that of exponentially or linearly decreasing law. It is in agreement with the microscopic finding for α decay and cluster radioactivity concerning the preformation of a cluster at the surface, which then penetrates by quantum tunneling the potential barrier.

This work was supported within the IDEI Programme under Contracts No. 43/05.10.2011 and 42/05.10.2011 with UEFISCDI, and NUCLEU Pro-

* poenaru@fias.uni-frankfurt.de

- [1] G. Gamow, Zur Quantentheorie des Atomkernes. Zeitschrift für Physik **51**, 204 (1928).
- [2] R.W. Gurney and E.U. Condon, Wave mechanics and radioactive disintegrations. Nature **122**, 439 (1928).
- [3] H.J. Rose and G.A. Jones, A new kind of natural radioactivity. Nature **307**, 245 (1984).
- [4] Encyclopaedia Britannica Online, 2011. Web <http://www.britannica.com/EBchecked/topic/465998/>.
- [5] R.G. Lovas, R.J. Liotta, A. Insolia, K. Varga, D.S. Delion, Microscopic theory of cluster radioactivity. Physics Reports **294**, 265 (1998).
- [6] C. Signarbieux *et al.*, Evidence for nucleon pair breaking even in the coldest scission configurations of ^{234}U and ^{236}U . Journal de Physique (Paris) Lettres **42**, L437 (1981).
- [7] V.M. Strutinsky, Shell effects in nuclear masses and deformation energies, Nuclear Physics, A **95**, 420 (1967).
- [8] A.M. Lane, Reduced widths of individual nuclear energy levels, Rev. Mod. Phys. **32**, 519 (1960).
- [9] D.N. Poenaru, M. Ivaşcu, and D. Mazilu, A new semiempirical formula for the alpha decay half-lives. J. Phys. Lettres **41**, L589 (1980).
- [10] Y. Wang, S. Wang, Z. Hou, and J. Gu, Systematic study of alpha-decay energies and half-lives of superheavy nuclei. Phys. Rev. C **92**, 064301 (2015).
- [11] N. Bohr, Neutron capture and nuclear constitution. Nature **137**, 344 (1936).
- [12] L. Meitner and O.R. Frisch, Disintegration of uranium by neutrons: a new type of nuclear reaction. Nature **143**, 239 (1939).
- [13] O. Hahn and F. Strassmann, Über den Nachweis und das Verhalten der bei der Bestrahlung des Urans mittels Neutronen entstehenden Erdalkalimetalle. Naturwissenschaften **27**, 11 (1939).
- [14] N. Bohr and J. Wheeler, The mechanism of nuclear fission. Phys. Rev. **56**, 426 (1939).
- [15] G.N. Flerov and K.A. Petrjak, Spontaneous fission of uranium. Phys. Rev. **58**, 89 (1940).
- [16] S.M. Polikanov *et al.*, Spontaneous fission with an anomalously short period. Soviet Physics JETP **15**, 1016 (1962).
- [17] *Particle Emission from Nuclei - 3 volumes*, (CRC Press, Boca Raton, Florida, 1989) Eds. D.N. Poenaru and M. Ivaşcu.
- [18] *Nuclear Decay Modes*, (Institute of Physics Publishing, Bristol, 1996) Ed. D.N. Poenaru.
- [19] W. Greiner and D.N. Poenaru, in *Encyclopedia of Condensed Matter Physics*, Vol. 5, (Elsevier, Oxford, 2005), Eds F. Bassani, G.L. Liedl, and P. Wyder, pp. 106–116.
- [20] J.H. Hamilton, S. Hofmann, and Y. Oganessian, Search for superheavy nuclei. Ann. Rev. Nucl. Part. Sci. **63**, 383 (2013).
- [21] Y.T. Oganessian, Sizing up the heavyweights. Nature **413**, 122 (2001).
- [22] D.N. Poenaru, R.A. Gherghescu, and W. Greiner, Heavy particle radioactivities of superheavy nuclei. Phys. Rev. Lett. **107**, 062503 (2011).
- [23] A. Sobiczewski, Theoretical description of superheavy nuclei. Radiochimica Acta **99**, 395 (2011).
- [24] A. Staszczak, A. Baran, and W. Nazarewicz, Spontaneous fission modes and lifetimes of superheavy elements in the nuclear density functional theory. Phys. Rev. C **87**, 024320 (2013).
- [25] M. Warda and L.M. Robledo, Microscopic description of cluster radioactivity in actinide nuclei. Phys. Rev. C **84**, 044608 (2011).
- [26] D.N. Poenaru and R.A. Gherghescu, Spontaneous fission of superheavy nucleus ^{286}Fl . Phys. Rev. C **94**, 014309 (2016).
- [27] M. Brack *et al.*, Funny hills: the shell correction approach to nuclear shell effects and its applications to the fission process. Rev. Mod. Phys. **44**, 320 (1972).
- [28] R.A. Gherghescu, Deformed two center shell model. Phys. Rev. C **67**, 014309 (2003).
- [29] H.J. Krappe, J.R. Nix, and A.J. Sierk, Unified nuclear potential for heavy-ion elastic scattering, fusion, fission and ground-state masses and deformations. Phys. Rev. C **20**, 992 (1979).
- [30] J. Bardeen, L. Cooper, and J. Schrieffer, Theory of superconductivity. Phys. Rev. C **108**, 1175 (1957).